this difference is that the boundary layer was too thin (0.02-0.08 cm). As a result, only a few particles were fully immersed in the boundary layer and the majority of the particles projected well outside. Moreover, the small particles which were fully immersed and could have been normally entrained due to lift are surrounded by larger ones and hence are protected to some extent from being displaced. This in turn necessitates larger surface dynamic pressures to dislodge them, resulting in a lower loading parameter. However, in practice, the VTOL configurations are many times larger than the 40.64 cm ducted fan and hence the boundary layer thickness can be such that bulk of the terrain particles can be fully immersed in it. Hence, the present criteria can be expected to give reasonably good results.

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Optimal Design of Ring Stiffened Cylindrical Shells Using Multiple Stiffener Sizes

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Introduction

KUNOO and Yang[1] investigated minimum weight cylindrical shells stiffened with both multiple size "I"-type rings and stringers using discrete stiffener buckling theory. They obtained a savings in weight of about 5% with the use of two ring and two stringer sizes for the example studied. Approximation methods are employed to reduce the computational effort required to approach a solution within reasonable bounds on cost (about 2000 s on a CDC 6500). Oddly, no mention is made in their work of the coalescence of buckling modes, a characteristic of optimal designs controlled by buckling behavior. Furthermore, their procedure apparently does not consider this situation.

This Note studies a "T"-type ring-stiffened shell problem by direct optimization without use of approximations or limitations in the number of stiffener sizes like those used in Ref. 1. The optimization formulation and procedure used here admit a large number of simultaneous buckling modes, thus allowing optimization under conditions of mode coalescence.

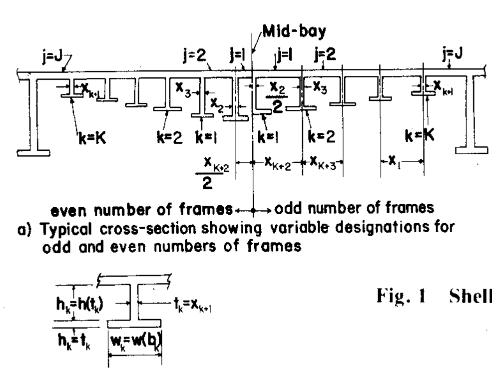
Procedure

The variables employed for this problem are skin thickness and ring dimensions and spacing. Each ring size used introduces a variable set associated with its dimensions. Thus, the number of variables is dependent on the number of sizes employed. To reduce problem dimensionality in this preliminary study, it is useful to introduce several assumptions. It is assumed that the shell is symmetrical with respect to a plane at midshell normal to the cylinder axis, that the ring flange thickness is equal to the web thickness, and that the flange width and web height are set at the maximum permitted to prevent local flange or web buckling.[2] It is shown in Ref. 2 that these assumptions have little effect on the optimal structural weight. The resulting problem variables as shown in Fig. 1.

Constraints on ring and skin yielding are applied in a fashion similar to Ref. 2 except that here a separate constraint must be considered for each shell panel. To determine the buckling pressures for the range of parameters of interest in this study, one could prudently examine all mode combinations where n (the number of circumferential waves) ranges from 0 to 20 and m (the number of axial half-waves) from 1 to 40. Using the analytical procedure of Refs. 3 and 4, this would involve the solution of several hundred eigenvalue problems of rank 840 (21×40) during a single optimization run. It may be seen, however, from an examination of the equations of Ref. 3 that for the case of uniform stiffeners the buckling modes are uncoupled with respect to n and interact only with respect to even or odd m. Thus, a single 840×840 problem can be reduced to a series of forty-two 20×20 problems substantially reducing computational effort required to determine buckling behavior. Furthermore, since most constraint function evaluations are for very similar designs, computational effort may be again reduced by restricting the range of odd or even m terms included in the formulation of the eigenvalue problem for a particular n based on a knowledge of the range necessary to include all m terms making a significant contribution. Likewise, only those n values which appear to be "critical" with respect to buckling need be examined.

Thus, let v_{mn}^s be the value of an element of the matrix of eigenvectors which represents the buckling behavior of the design x^s, and m_n^s be the value of m associated with the largest value of the component v_{mn}^s of vector v_n^s. Now, using the procdure of Ref. 3 starting from $n = n_{\min}$, where $n_{\min}$ is the lowest n considered, and setting an index $i = 1$, set up and solve an M by M eigenvalue problem P_n^s for design x^s using terms associated with

$$m = m_{\min}, \quad m_{\min} + 2, \quad m_{\min} + 4, \ldots, m_{\max} \tag{1}$$



a) Typical cross-section showing variable designations for odd and even numbers of frames

b) Dimensions of the k-th frame

Fig. 1 Shell design variables.

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where all m are odd. Here,

$$m_{\max} = m_{\min} + 2(M-1) \tag{2}$$

where M is the number of m terms used for the analysis, and

$$m_{\min} = \begin{cases} 1, & m_n^{s-1} \le M \\ m_n^{s-1} - M, & m_n^{s-1} \ge M \end{cases} \tag{3}$$

where m_n^{s-1} is the m of the largest component of its associated nth eigenvector for the last design x^{s-1}. Now, if

$$m_n^s = m_n^{s-1} \tag{4}$$

or

$$m_n^s < M \tag{5}$$

it means that for a given number of terms M, the range for the above problem was properly placed, and thus the problem P_n^s was the "best" problem. On the other hand, if one of these conditions is not met, a new problem P_n^* is formulated per Eqs. (1-3) where m_n^s replaces m_n^{s-1}. If conditions (4) or (5) are now satisfied where m_n^* replaces m_n^s and m_n^s replaces m_n^{s-1} in these equations, then problem P_n^* is the "best" problem. If not, the process is repeated until conditions (4) or (5) are satisfied or oscillation is detected whereupon that problem of the last three solved producing the lowest eigenvalue is taken as the best problem. The m_n^* or m_n^s associated with the best problem, which is now called P_n^s, is then called m_n^s and used at the next design iteration x^{s+1}.

The first T eigenvalues of λ_{tn}^s, $t = 1,2 \ldots T$ (which are the collapse pressures) of this problem are used to form r constraints for this n where m are odd. This process is repeated for this n and even m. Constraints are then evaluated in a similar fashion for all n to be examined.

It should be noted that the treatment of buckling in this formulation is substantially different than that used in earlier optimization studies using orthotropic shell theory. These use only two buckling constraints, one for general and one for shell or panel (interframe) buckling.[2,5] Here, all modes need to be examined and constraints established for all those that may be active, since any attempt made to lighten a design while moving to avoid the constraints associated with only two buckling modes may produce a violation in some other mode. This situation is analogous to the frequency separation problem discussed in Ref. 5.

To calculate the buckling constraint derivatives needed by the optimization procedure[6] at a point x^s for constraints

Table 1 Optimal shell designs

		3 Rings		11 Rings		19 Rings	
		Identical	Multiple	Identical	Multiple	Identical	Multiple
Weight/displacement ratio[2]		0.220	0.219	0.140	0.138	0.112	0.105
Skin thickness, mm		68.15	68.15	39.45	38.61	29.27	26.95
(in.)		(2.683)	(2.683)	(1.553)	(1.520)	(1.113)	(1.061)
Ring thickness, mm	1[a]	17.27	20.37	14.78	16.31	13.67	15.52
(in.)		(0.680)	(0.802)	(0.582)	(0.642)	(0.538)	(0.611)
	2	17.27	13.23	↑	15.62	↑	7.93
		(0.680)	(0.521)		(0.615)		(0.312)
	3	—	—		13.08		16.94
					(0.515)		(0.667)
	4				13.11		7.83
					(0.516)		(0.308)
	5	—	—	↓	16.36		13.69
					(0.644)		(0.539)
	6			14.78	16.41		16.08
				(0.582)	(0.646)		(0.633)
	7	—	—	—	—		9.17
							(0.361)
	8						15.57
							(0.613)
	9					↓	7.70
							(0.303)
	10					13.67	15.82
						(0.538)	(0.643)
Ring spacing, m	1[a]	3.772	3.813	1.257	1.259	0.7544	0.7573
(in.)		(148.5)	(150.1)	(49.50)	(49.56)	(29.70)	(29.82)
	2	3.772	3.729	↑	1.259	↑	0.7579
		(148.5)	(146.8)		(49.56)		(29.84)
	3	—	—		1.259		0.7554
					(49.56)		(29.74)
	4				1.259		0.7532
					(49.56)		(29.66)
	5	—	—	↓	1.259		0.7522
					(49.56)		(29.615)
	6			1.257	1.249		0.7417
				(49.50)	(49.18)		(29.20)
	7	—	—	—	—		0.7515
							(29.58)
	8					↓	0.7487
							(29.48)
	9					0.7544	0.7489
						(29.70)	(29.49)
n values of active buckling modes		4,8,9	4,6,8,9	3,13,14,15	3,12,13,14,15	3,11	3,9,10,11
m values of active buckling modes		1,4	1,2,4	1,12	1,9,12	1,20,22	1,2,4,6,8,20

[a] Ring or ring spacing number counting from the center of the shell outward.

derived from a particular eigenvalue problem P_n^s, a similar problem P_n^{si} is formed using $m_n^{si} = m_n^s$ and solved using $x^{si} = x^s + \Delta x_i$ is a small change in the ith coordinate direction of vector x^s. The lowest T eigenvalues of this problem are then used to compute the ith components of the T derivatives associated with the constraints derived from P_n^s, where the lowest eigenvalue of problem P_n^{si} is associated with the lowest of P_n^s to estimate the derivative of the lowest eigenvalue. The second lowest eigenvalue of P_n^{si} is associated with the second lowest of P_n^s, etc.

Results

The 304.8-m (1000-ft) immersion depth study of Ref. 2 was repeated here using steel with an allowable skin and ring stress of 620.4 MPa (90,000 psi). For this study, shell radius of 5.029 m (198 in.), shell length 15.09 m (594 in.), $\gamma_w = 1.0256$ g/cm^3 (0.0374 lb/in.^{3}), $\gamma_s = 7.733$ g/cm^3 (0.282 lb/in.^{3}), $E = 20.68 \times 10^4$ MPa (30×10^6 psi), and $\mu = 0.25$ are used. The n modes from $n=3$ to $n=16$ were found to contain all active constraints and were investigated at all design points. A single buckling constraint ($T=1$) was used for the n modes with odd m and $n \leq 5$ with even n. The problem formulations for these cases used seven m terms ($M=7$). For $n>5$ with even m terms, two constraints ($T=2$) were generated for each n mode and fifteen m terms were used ($M=15$).

Three sets of two optimization runs were made using 3, 11, and 19 frames. In the first run, equally spaced equal ring sizes were employed. The optimal equal size ring configuration was then used to start the second run where the ring sizes and spacings were allowed to vary.

The results are summarized in Table 1. All designs are constrained only by buckling. The 19-ring problem required about 3000-s CPU time on an IBM 370/158 using the H-level compiler. Consider first the shell reinforced with only three rings. The design using identical rings is, as expected,[2] controlled by buckling modes where $m=1$ and 4, the general and shell buckling modes. In the design using multiple ring sizes, it may be seen that, as expected, the center ring is largest in order to suppress a mode where $n=6$ and $m=2$ which became active as the ring nearest the ends was reduced in size in an effort to reduce weight.

Skin thickness is controlled in both designs by the $n=8$ and $m=4$ modes. Thus, the two designs have identical skin thicknesses since for this configuration the torsional stiffness of the rings does not significantly effect these modes. Now, since the skin represents most of the weight of the shell segment in these designs, the small improvement in ring efficiency produces a negligible savings in overall weight (about 0.45%).

Now, consider the shells using nineteen rings. Here, the use of multiple ring sizes saves about 5½% in weight. The design using identical frames is controlled, as expected, by buckling modes where $m=1$ and $m=20$ are dominant. The use of multiple ring sizes allows redistribution of ring material to help suppress these modes. This redistribution occurs until modes where $m=1,2,4,6,8$, and 20 all control the design. Further significant improvement in ring material distribution then becomes impossible. In this case, the skin thickness is reduced significantly by ring material redistribution because of the relatively short panel segments and the relatively large ring to skin thickness ratios. The torsional stiffness of the ring under such conditions is important in the panel buckling mode behavior. Thus, redistribution of ring material can effectively be used to suppress such modes where rings are closely spaced. The skin thickness in such designs using multiple ring sizes can therefore be significantly thinner than in optimal designs using identical ring sizes. The combined effects of savings in the weight of both rings and skin then produce significant overall weight reduction.

Conclusion

This mode coalescence observed has significant implications in analysis since it raises the question of the adequacy of design buckling criteria. Most failure criteria are based on study of a single failure mode, and thus ignore interaction between modes. Their accuracy and safety under conditions where several modes are active is therefore suspect since one would expect interactions between failure modes to produce a reduction in structural strength. Thus, for optimal designs to be used with confidence, existing failure criteria must be validated under simultaneously active mode conditions, or, more likely, criteria considering mode interaction must be developed.

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